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Constant-Cutoff Approach to Hyperon Polarizabilities in the Bound-State Soliton Model

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We suggest a quantum stabilization method for the $SU(2)$ σ-model, based on the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.*, which avoids the difficulties with the usual soliton boundary conditions pointed out by Iwasaki and Ohyama. We investigate the baryon number $B = 1$ sector of the model and show that after the collective coordinate quantization it admits a stable soliton solution which depends on a single dimensional arbitrary constant. We then apply this approach to the calculation of electric and magnetic static polarizabilities of octet hyperons in the bound-state $SU(3)$-soliton model for hyperons, with $SU(3)$-symmetry breaking. The results, with both seagull and dispersive contributions included, are compared with the predictions obtained using the complete Skyrme model.

1. INTRODUCTION

It was shown by Skyrme (1961, 1962) that baryons can be treated as solitons of a nonlinear chiral theory. The original Lagrangian of the chiral $SU(2)$ σ-model is

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} \partial_\mu U \, \partial^\mu U^+ \tag{1.1}$$

where

$$U = \frac{2}{F_\pi} (\sigma + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}) \tag{1.2}$$

is a unitary operator ($UU^+ = 1$) and F_π is the pion-decay constant. In (1.2) $\sigma = \sigma(\mathbf{r})$ is a scalar meson field and $\boldsymbol{\pi} = \boldsymbol{\pi}(\mathbf{r})$ is the pion isotriplet.

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The classical stability of the soliton solution to the chiral σ-model Lagrangian requires an additional ad hoc term, proposed by Skyrme (1961, 1962), to be added to (1.1),

$$\mathcal{L}_{Sk} = \frac{1}{32e^2} \operatorname{Tr}[U^+\partial_\mu U, U^+\partial_\nu U]^2 \tag{1.3}$$

with a dimensionless parameter e and where $[A, B] = AB - BA$. It was shown by several authors [Adkins *et al.* (1983); see also Witten *et al.* (1979, 1983a, b); for an extensive list of other references see Holzwarth and Schwesinger (1986), Nyman and Riska (1990)] that, after collective quantization using the spherically symmetric ansatz

$$U_0(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.4}$$

the chiral model, with both (1.1) and (1.3) included, gives good agreement with experiment for several important physical quantities. Thus it should be possible to derive the effective chiral Lagrangian, obtained as a sum of (1.1) and (1.3), from a more fundamental theory like QCD. On the other hand it is not easy to generate a term like (1.3) and give a clear physical meaning to the dimensionless constant e in (1.3) using QCD.

Mignaco and Wulck (1989) (MW) indicated therefore the possibility to build a stable single-baryon ($n = 1$) quantum state in the simple chiral theory with the Skyrme stabilizing term (1.3) omitted. MW showed that the chiral angle $F(r)$ is in fact a function of a dimensionless variable $s = \frac{1}{2}\chi''(0)r$, where $\chi''(0)$ is an arbitrary dimensional parameter intimately connected to the usual stability argument against the soliton solution for the nonlinear σ-model Lagrangian.

Using the adiabatically rotated ansatz $U(\mathbf{r}, t) = A(t)U_0(\mathbf{r})A^+(t)$, where $U_0(\mathbf{r})$ is given by (1.4), MW obtained the total energy of the nonlinear σ-model soliton in the form

$$E = \frac{\pi}{4} F_\pi^2 \frac{1}{\chi''(0)} a + \frac{1}{2} \frac{[\chi''(0)]^3}{(\pi/4)F_\pi^2 b} J(J + 1) \tag{1.5}$$

where

$$a = \int_0^\infty \left[\frac{1}{4} s^2 \left(\frac{d\mathcal{F}}{ds}\right)^2 + 8 \sin^2\left(\frac{1}{4} \mathcal{F}\right)\right] ds \tag{1.6}$$

$$b = \int_0^\infty ds \frac{64}{3} s^2 \sin^2\left(\frac{1}{4} \mathcal{F}\right) \tag{1.7}$$

and $\mathcal{F}(s)$ is defined by

$$F(r) = F(s) = -n\pi + \tfrac{1}{4}\mathcal{F}(s) \tag{1.8}$$

The stable minimum of the function (1.5) with respect to the arbitrary dimensional scale parameter $\chi''(0)$ is

$$E = \frac{4}{3} F_\pi \left[\frac{3}{2}\left(\frac{\pi}{4}\right)^2 \frac{a^3}{b} J(J+1)\right]^{1/4} \tag{1.9}$$

Despite the nonexistence of the stable classical soliton solution to the nonlinear σ-model, it is possible, after the collective coordinate quantization, to build a stable chiral soliton at the quantum level, provided that there is a solution $F = F(r)$ which satisfies the soliton boundary conditions, i.e., $F(0) = -n\pi$, $F(\infty) = 0$, such that the integrals (1.6) and (1.7) exist.

However, as pointed out by Iwasaki and Ohyama (1989), the quantum stabilization method in the form proposed by Mignaco and Wulck (1989) is not correct, since in the simple σ-model the conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. In other words, if the condition $F(0) = -\pi$ is satisfied, Iwasaki and Ohyama obtained numerically $F(\infty) \to -\pi/2$, and the chiral phase $F = F(r)$ with correct boundary conditions does not exist.

Iwasaki and Ohyama also proved analytically that both boundary conditions $F(0) = -n\pi$ and $F(\infty) = 0$ cannot be satisfied simultaneously. Introducing a new variable $y = 1/r$ into the differential equation for the chiral angle $F = F(r)$, we obtain

$$\frac{d^2F}{dy^2} = \frac{1}{y^2} \sin 2F \tag{1.10}$$

There are two kinds of asymptotic solutions to equation (1.10) around the point $y = 0$, which is called a regular singular point if $\sin 2F \approx 2F$. These solutions are

$$F(y) = \frac{m\pi}{2} + cy^2, \qquad m = \text{even integer} \tag{1.11}$$

$$F(y) = \frac{m\pi}{2} + \sqrt{cy} \cos\left[\frac{\sqrt{7}}{2} \ln(cy) + \alpha\right], \qquad m = \text{odd integer} \tag{1.12}$$

where c is an arbitrary constant and α is a constant to be chosen appropriately. When $F(0) = -n\pi$, then we want to know which of these two solutions are approached by $F(y)$ when $y \to 0$ ($r \to \infty$). In order to answer that question we multiply (1.10) by $y^2F'(y)$, integrate with respect to y from y to ∞, and use $F(0) = -n\pi$. Thus we get

$$y^2F'(y) + \int_y^\infty 2y[F'(y)]^2\, dy = 1 - \cos[2F(y)] \tag{1.13}$$

Since the left-hand side of (1.13) is always positive, the value of $F(y)$ is always limited to the interval $n\pi - \pi < F(y) < n\pi + \pi$. Taking the limit $y \to 0$, we find that (1.13) is reduced to

$$\int_0^\infty 2y[F'(y)]^2\, dy = 1 - (-1)^m \tag{1.14}$$

where we used (1.11)–(1.12). Since the left-hand side of (1.14) is strictly positive, we must choose an odd integer m. Thus the solution satisfying $F(0) = -n\pi$ approaches (1.12) and we have $F(\infty) \neq 0$. The behavior of the solution (1.11) in the asymptotic region $y \to \infty$ ($r \to 0$) is investigated by multiplying (1.10) by $F'(y)$, integrating from 0 to y, and using (1.11). The result is

$$[F'(y)]^2 = \frac{2\sin^2 F(y)}{y^2} + \int_0^y \frac{2\sin^2 F(y)}{y^3}\, dy \tag{1.15}$$

From (1.15) we see that $F'(y) \to$ const as $y \to \infty$, which means that $F(r) \simeq 1/r$ for $r \to 0$. This solution has a singularity at the origin and cannot satisfy the usual boundary condition $F(0) = -n\pi$.

In Dalarsson (1991a, b, 1992), I suggested a method to resolve this difficulty by introducing a radial modification phase $\varphi = \varphi(r)$ in the ansatz (1.4) as follows:

$$U(\mathbf{r}) = \exp[i\boldsymbol{\tau}\cdot\mathbf{r}_0 F(r) + i\varphi(r)], \qquad \mathbf{r}_0 = \mathbf{r}/r \tag{1.16}$$

Such a method provides a stable chiral quantum soliton, but the resulting model is an entirely noncovariant chiral model, different from the original chiral σ-model.

In the present paper we use the constant-cutoff limit of the cutoff quantization method developed by Balakrishna *et al.* (1991; see also Jain *et al.*, 1989) to construct a stable chiral quantum soliton within the original chiral σ-model. Then we apply this method to calculate electric and magnetic static polarizabilities of octet hyperons in the bound-state $SU(3)$-soliton model for hyperons, with $SU(3)$-symmetry breaking. The results, with both seagull and dispersive contributions included, are compared with the predictions obtained using the complete Skyrme model (Gobbi *et al.*, 1996), showing that there is a general qualitative agreement between our results and the results of the complete Skyrme model (Gobbi *et al.*, 1996).

The reason the cutoff approach to the problem of the chiral quantum soliton works is connected to the fact that the solution $F = F(r)$, which satisfies the boundary condition $F(\infty) = 0$, is singular at $r = 0$. From the physical point of view the chiral quantum model is not applicable to the region about the origin, since in that region there is a quark-dominated bag of the soliton.

However, as argued in Balakrishna *et al.* (1991), when a cutoff ϵ is introduced, the boundary conditions $F(\epsilon) = -n\pi$ and $F(\infty) = 0$ can be satisfied. Balakrishna *et al.* (1991) discuss an interesting analogy with the damped pendulum, showing clearly that as long as $\epsilon > 0$, there is a chiral phase $F = F(r)$ satisfying the above boundary conditions. The asymptotic forms of such a solution are given by Eq. (2.2) in Balakrishna *et al.* (1991). From these asymptotic solutions we immediately see that for $\epsilon \to 0$ the chiral phase diverges at the lower limit.

Different applications of the constant-cutoff approach have been been discussed in Dalarsson (1993, 1995a–d, 1996a–c, 1997).

2. CONSTANT-CUTOFF STABILIZATION

Substituting (1.4) into (1.1), we obtain for the static energy of the chiral baryon

$$E_0 = \frac{\pi}{2} F_\pi^2 \int_{\epsilon(t)}^{\infty} dr \left[r^2 \left(\frac{dF}{dr} \right)^2 + 2 \sin^2 F \right] \tag{2.1}$$

In (2.1) we avoid the singularity of the profile function $F = F(r)$ at the origin by introducing the cutoff $\epsilon(t)$ at the lower boundary of the space interval $r \in [0, \infty]$, i.e., by working with the interval $r \in [\epsilon, \infty]$. The cutoff itself is introduced following Balakrishna *et al.* (1991) as a dynamic time-dependent variable.

From (2.1) we obtain the following differential equation for the profile function $F = F(r)$:

$$\frac{d}{dr}\left(r^2 \frac{dF}{dr} \right) = \sin 2F \tag{2.2}$$

with the boundary conditions $F(\epsilon) = -\pi$ and $F(\infty) = 0$, such that the correct soliton number is obtained. The profile function $F = F[r; \epsilon(t)]$ now depends implicitly on time t through $\epsilon(t)$. Thus in the nonlinear σ-model Lagrangian

$$L = \frac{F_\pi^2}{16} \int \mathrm{Tr}(\partial_\mu U \, \partial^\mu U^+) \, d^3\mathbf{r} \tag{2.3}$$

we use the ansätze

$$U(\mathbf{r}, t) = A(t) U_0(\mathbf{r}, t) A^+(t), \qquad U^+(\mathbf{r}, t) = A(t) U_0^+(\mathbf{r}, t) A^+(t) \tag{2.4}$$

where

$$U_0(\mathbf{r}, t) = \exp\{i\boldsymbol{\tau}\cdot\mathbf{r}_0 F[r; \epsilon(t)]\} \tag{2.5}$$

The static part of the Lagrangian (2.3), i.e.,

$$L = \frac{F_\pi^2}{16}\int \mathrm{Tr}(\nabla U\cdot\nabla U^+)\, d^3\mathbf{r} = -E_0 \tag{2.6}$$

is equal to minus the energy E_0 given by (2.1). The kinetic part of the Lagrangian is obtained using (2.4) with (2.5), and is equal to

$$L = \frac{F_\pi^2}{16}\int \mathrm{Tr}(\partial_0 U\; \partial_0 U^+)\, d^3\mathbf{r} = bx^2\, \mathrm{Tr}[\partial_0 A\; \partial_0 A^+] + c[\dot{x}(t)]^2 \tag{2.7}$$

where

$$b = \frac{2\pi}{3} F_\pi^2 \int_1^\infty \sin^2 F\, y^2 dy, \qquad c = \frac{2\pi}{9} F_\pi^2 \int_1^\infty y^2\left(\frac{dF}{dy}\right)^2 y^2\, dy \tag{2.8}$$

with $x(t) = [\epsilon(t)]^{3/2}$ and $y = r/\epsilon$. On the other hand, the static energy functional (2.1) can be rewritten as

$$E_0 = ax^{2/3}, \qquad a = \frac{\pi}{2} F_\pi^2 \int_1^\infty \left[y^2\left(\frac{dF}{dy}\right)^2 + 2\sin^2 F\right] dy \tag{2.9}$$

Thus the total Lagrangian of the rotating soliton is given by

$$L = c\dot{x}^2 - ax^{2/3} + 2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu \tag{2.10}$$

where $\mathrm{Tr}(\partial_0 A\;\partial_0 A^+) = 2\dot{\alpha}_\nu\dot{\alpha}^\nu$ and α_ν $(\nu = 0, 1, 2, 3)$ are the collective coordinates defined as in Bhaduri (1988). In the limit of a time-independent cutoff ($\dot{x} \to 0$) we can write

$$H = \frac{\partial L}{\partial\dot{\alpha}^\nu}\dot{\alpha}^\nu - L = ax^{2/3} + 2bx^2\dot{\alpha}_\nu\dot{\alpha}^\nu = ax^{2/3} + \frac{1}{2bx^2}J(J+1) \tag{2.11}$$

where $\langle \mathbf{J}^2\rangle = J(J+1)$ is the eigenvalue of the square of the soliton angular momentum. A minimum of (2.11) with respect to the parameter x is reached at

$$x = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{-3/8} \Rightarrow \epsilon^{-1} = \left[\frac{2}{3}\frac{ab}{J(J+1)}\right]^{1/4} \tag{2.12}$$

The energy obtained by substituting (2.12) into (2.11) is given by

$$E = \frac{4}{3}\left[\frac{3}{2}\frac{a^3}{b}J(J+1)\right]^{1/4} \tag{2.13}$$

This result is identical to the result obtained by Mignaco and Wulck, which is easily seen if we rescale the integrals a and b in such a way that $a \rightarrow (\pi/4)F_\pi^2 a$ and $b \rightarrow (\pi/4)F_\pi^2 b$ and introduce $f_\pi = 2^{-3/2}F_\pi$. However, in the present approach, as shown in Balakrishna *et al.* (1991), there is a profile function $F = F(y)$ with proper soliton boundary conditions $F(1) = -\pi$ and $F(\infty) = 0$ and the integrals a, b, and c in (2.9)–(2.10) exist and are shown in Balakrishna *et al.* (1991) to be $a = 0.78$ GeV^2, $b = 0.91$ GeV^2, and $c = 1.46$ GeV^2 for $F_\pi = 186$ MeV.

Using (2.13), we obtain the same prediction for the mass ratio of the lowest states as Mignaco and Wulck (1989), which agrees rather well with the empirical mass ratio for the Δ-resonance and the nucleon. Furthermore, using the calculated values for the integrals a and b, we obtain the nucleon mass $M(N) = 1167$ MeV, which is about 25% higher than the empirical value of 939 MeV. However if we choose the pion-decay constant equal to $F_\pi = 150$ MeV, we obtain $a = 0.507$ GeV^2 and $b = 0.592$ GeV^2, giving the exact agreement with the empirical nucleon mass.

Finally, it is of interest to know how large the constant cutoffs are for the above values of the pion-decay constant in order to che4ck if they are in the physically acceptable ballpark. Using (2.12), it is easily shown that for the nucleons ($J = 1/2$) the cutoffs are equal to

$$\epsilon = \begin{cases} 0.22\ \text{fm} & \text{for} \quad F_\pi = 186\ \text{MeV} \\ 0.27\ \text{fm} & \text{for} \quad F_\pi = 150\ \text{MeV} \end{cases} \tag{2.14}$$

From (2.14) we see that the cutoffs are too small to agree with the size of the nucleon (0.72 fm), as we should expect, since the cutoffs rather indicate the size of the quark-dominated bag in the center of the nucleon. Thus we find that the cutoffs are of reasonable physical size. Since the cutoff is proportional to F_π^{-1}, we see that the pion-decay constant must be less than 57 MeV in order to obtain a cutoff which exceeds the size of the nucleon. Such values of pion-decay constant are not relevant to any physical phenomena.

3. THE *SU*(3)-EXTENDED CONSTANT-CUTOFF MODEL

3.1. The Effective Interaction

The Lagrangian density for the bound-state model of hyperons is, with Skyrme stabilizing term omitted and in the presence of the electromagnetic interaction, given by (Dalarsson, 1993, 1995a–d, 1996a–c, 1997; Bhaduri, 1988)

$$\mathcal{L} = \frac{F_\pi^2}{16} \operatorname{Tr} D_\mu U\, D^\mu U^+ + \frac{F_\pi^2}{16} m_\pi^2 \operatorname{Tr}(U + U^+ - 2)$$
$$- \frac{1}{48} (F_K^2 - F_\pi^2) \operatorname{Tr}(1 - \sqrt{3}\lambda_8)(U\, D_\mu U\, D^\mu U^+ + D_\mu U\, D^\mu U^+ U^+)$$
$$+ \frac{1}{24} (F_K^2 m_K^2 - F_\pi^2 m_\pi^2) \operatorname{Tr}(1 - \sqrt{3}\lambda_8)(U + U^+ - 2) \qquad (3.1)$$

where m_π and m_K are pion and kaon masses, respectively, and F_K is the kaon weak-decay constant, with the empirical ratio to pion decay constant $F_K/F_\pi \approx 1.23$. The first term in (3.1) is the usual σ-model Lagrangian, while the remaining three terms are all chiral- and flavor-symmetry-breaking terms, present in the mesonic sector of the model. All flavor-symmetry-breaking terms in the effective Lagrangian (3.1) also break the chiral symmetry just as quark-mass terms do in the underlying QCD Lagrangian.

In (3.1) the covariant derivatives are defined by

$$\partial_\mu U \to D_\mu U = \partial_\mu U + ieA_\mu[Q, U] \qquad (3.2)$$

$$\partial_\mu U^+ \to D_\mu U^+ = \partial_\mu U^+ + ieA_\mu[Q, U^+] \qquad (3.3)$$

where A_μ is the electromagnetic field, e is the elementary electric charge, Q is the electric charge operator defined by

$$Q = \frac{1}{2}\left(\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8\right) \qquad (3.4)$$

and λ_i ($i = 1, \ldots, 8$) are standard $SU(3)$ matrices.

In addition to the action obtained using the Lagrangian (3.1), the Wess–Zumino action, with the electromagnetic interaction, of the form

$$S^{wz} = -\frac{iN_c}{240\pi^2} \int d^5x\, e^{\mu\nu\alpha\beta\gamma} \operatorname{Tr}[U^+\partial_\mu U\, U^+\partial_\nu U\, U^+\partial_\alpha U\, U^+\partial_\beta U\, U^+\partial_\gamma U]$$
$$- \frac{N_c}{48\pi^2} \int d^4x\, e^{\mu\nu\alpha\beta} \{eA_\mu \operatorname{Tr}[Q(U^+\partial_\nu U\, U^+\partial_\alpha U\, U^+\partial_\beta U$$
$$- U\partial_\nu U^+\, U\partial_\alpha U^+\, U\partial_\beta U^+)]$$
$$- ie^2 A_\mu \partial_\nu A_\alpha \operatorname{Tr}[2Q^2(U^+\partial_\beta U - U\partial_\beta U^+) + QU^+QU\, U^+\partial_\beta U$$
$$- QUQU^+\, U\partial_\beta U^+]\} \qquad (3.5)$$

must be included in the total action of the system, where N_c is the number of colors in the underlying QCD. The Wess–Zumino action defines the topological properties of the model important for the quantization of the solitons. In the $SU(2)$ case the Wess–Zumino action vanishes identically and was therefore not present in the discussions of Sections 1 and 2.

Using (3.1) and (3.5), one can write the total action, following Dalarsson (1993, 1995a–d, 1996a–c, 1997), as

$$S = S^{(0)} + \int d^4x\, eA_\mu J^\mu - \int d^4x\, e^2 A_\mu G^{\mu\nu} A_\nu \tag{3.6}$$

where

$$\begin{aligned} J^\mu = {} & \frac{iF_\pi^2}{8} \mathrm{Tr}[Q(U^+\partial^\mu U + U\partial^\mu U^+)] + \frac{i}{48}(F_K^2 - F_\pi^2) \\ & \times \mathrm{Tr}\{(1 - \sqrt{3}\lambda_8)([U, Q]U^+\partial^\mu U - U^+\partial^\mu U[U^+, Q] \\ & + [U^+, Q]U\partial^\mu U^+ - U\partial^\mu U^+[U, Q])\} \\ & - \frac{N_c}{48\pi^2} \epsilon^{\mu\nu\alpha\beta}\, \mathrm{Tr}[Q(U^+\partial_\nu U\, U^+\partial_\alpha U\, U^+\partial_\beta U - U\partial_\nu U^+\, U\partial_\alpha U^+\, U\partial_\beta U^+)] \end{aligned} \tag{3.7}$$

$$\begin{aligned} G^{\mu\nu} = {} & g^{\mu\nu}\biggl(\frac{F_\pi^2}{16} \mathrm{Tr}(Q - U^+QU)^2 + \frac{1}{48}(F_K^2 - F_\pi^2) \\ & \times \mathrm{Tr}\{(1 - \sqrt{3}\lambda_8)[(Q - U^+QU)^2U^+ + U(Q - U^+QU)^2]\}\biggr) \\ & + \frac{iN_c}{48\pi^2} \epsilon^{\mu\nu\alpha\beta}\, \mathrm{Tr}[(2Q^2 + QU^+QU)U^+\partial_\alpha U - (2Q^2 + QU^+QU)U\partial_\alpha U^+]\partial_\beta \end{aligned} \tag{3.8}$$

In (3.6) $S^{(0)}$ is the action in the absence of the electromagnetic field, and in the following it will be treated according to the usual approach to the constant-cutoff approach to the bound-state soliton model found in Dalarsson (1993, 1995a–d, 1996a–c, 1997). The meson–soliton field is written in the form

$$U = \sqrt{U_\pi}\, U_K \sqrt{U_\pi} \tag{3.9}$$

where U_π is the $SU(3)$-extension of the usual $SU(2)$ skyrmion field used to describe the nucleon spectrum, and U_K is the field describing the kaons

$$U_\pi = \begin{bmatrix} u_\pi & 0 \\ 0 & 1 \end{bmatrix}, \qquad U_K = \exp\left\{ i\frac{2^{3/2}}{F_\pi} \begin{bmatrix} 0 & K \\ K^+ & 0 \end{bmatrix} \right\} \tag{3.10}$$

In (3.4), u_π is the usual $SU(2)$-skyrmion field, given by (1.4). The two-dimensional vector K in (3.10) is the kaon doublet

$$K = \begin{bmatrix} K^+ \\ K^0 \end{bmatrix}, \qquad K^+ = [K^- \quad \bar{K}^0] \tag{3.11}$$

3.2. The Hyperon Spectrum

We now substitute (3.9), with U_π and U_K defined by (3.10), into the action $S^{(0)}$ of the kaon–soliton system and expand U_K to second order in kaon fields (3.11) to obtain the effective interaction Lagrangian density for the kaon–soliton system:

$$\mathcal{L} = \dot{K}^+\dot{K} + K^+\nabla^2 K + i\lambda(r)(K^+\dot{K} - \dot{K}^+K) - m_K^2 K^+K$$
$$- K^+ 2\,\frac{1-\cos F}{r^2}\,\mathbf{I}\cdot\mathbf{L}K + K^+ v_0(r)K \tag{3.12}$$

where **L** is the kaon orbital momentum and **I** is the total angular momentum of the rotating soliton. The term proportional to $\mathbf{I}\cdot\mathbf{L}$ represents the kaon–soliton (iso)spin–orbit interaction. In (3.12) we introduced the quantities $\lambda(r)$ and $v_0(r)$ as follows:

$$\lambda(r) = -\frac{N_c}{2\pi^2 F_K^2}\,\frac{\sin^2 F}{r^2}\,\frac{dF}{dr} \tag{3.13}$$

$$v_0 = \frac{1}{4}\left(\frac{dF}{dr}\right)^2 + \frac{\cos F\,(1-\cos F)}{r^2} + \frac{F_\pi^2 m_\pi^2}{2F_K^2}(1-\cos F) \tag{3.14}$$

The Hamiltonian density corresponding to the Lagrangian density (3.12) is given by

$$\mathcal{H} = \Pi^+\Pi - K^+\nabla^2 K - i\lambda(r)(K^+\Pi - \Pi^+K) \pm m_K^2 K^+K$$
$$+ K^+\left[2\,\frac{1-\cos F}{r^2}\,\mathbf{I}\cdot\mathbf{L} - v_0(r)\right]K + \lambda^2(r)K^+K \tag{3.15}$$

The kaon field (3.11) may be decomposed into modes with strangeness number $S = \pm 1$ as (Dalarsson, 1993, 1995a–d, 1996a–c, 1997)

$$K = \sum_m [\bar{K}_m(\mathbf{r})e^{i\omega_m t}\hat{b}_m^+ + K_m(\mathbf{r})e^{-i\omega_m t}\hat{a}_m] \tag{3.16}$$

with $\hat{a}_m$ and $\hat{b}_m^+$ being annihilation and creation operators for $S = -1$ and $S = +1$ modes, respectively. From (3.12) we obtain the wave equation for the $S = -1$ mode wave functions $K_m(r)$

$$\nabla^2 K_m(\mathbf{r}) + \left[v_0(r) - 2\,\frac{1-\cos F}{r^2}\,\mathbf{I}\cdot\mathbf{L}\right]K_m(\mathbf{r}) - m_K^2 K_m(\mathbf{r})$$
$$+ 2\omega_m\lambda(r)K_m(\mathbf{r}) + \omega_m^2 K_m(\mathbf{r}) = 0 \tag{3.17}$$

where the commutation rules for creation and annihilation operators in (3.16) give the orthonormality condition for wave functions K_m in the form

$$\int d^3\mathbf{r}\,[\omega_m + \omega_n + 2\lambda(r)]K_n^* K_m = \delta_{mn} \tag{3.18}$$

Expanding the kaon wave functions $K_m(\mathbf{r})$ in terms of vector spherical harmonics (Dalarsson, 1993, 1995a–d, 1996a–c, 1997)

$$K(\mathbf{r}) = \sum_{\alpha,L} k_{\alpha L}(r) Y_{\alpha IL} \tag{3.19}$$

the wave equation (3.14) becomes a one-dimensional differential equation which can be found in Dalarsson (1993, 1995a–d, 1996a–c, 1997).

In order to calculate the hyperon spectrum we must take into account the rotational modes of the soliton (Dalarsson, 1993, 1995a–d, 1996a–c, 1997). The kaon and soliton fields are rotated according to

$$K \rightarrow a(t)K \tag{3.20}$$

$$U \rightarrow A(t)UA^{+}(t) \tag{3.21}$$

where

$$A(t) = \begin{bmatrix} a(t) & 0 \\ 0 & 1 \end{bmatrix} \tag{3.22}$$

is an $SU(2)$ subgroup of $SU(3)$. The $SU(2)$ rotation operator $A(t)$ adds extra time-derivative terms to the Lagrangian. Using now the constant-cutoff stabilization procedure following Dalarsson (1993, 1995a–d, 1996a–c, 1997), we obtain the hyperon spectrum as follows:

$$E = \omega|S| + \frac{4}{3}\left\{\frac{3}{2}\frac{a^3}{b}[cJ(J+1) + (1-c)I(I+1) + \frac{1}{4}c(c-1)|S|(|S|+2)]\right\}^{1/4} \tag{3.23}$$

where I, J, and S are the isospin, spin, and strangeness hyperon quantum numbers, respectively.

3.3. Static Electric Polarizability

The static electric polarizability is most easily extracted from the result for the shift in soliton energy in an external constant electric field $\mathcal{E} = \mathcal{E}\mathbf{z}_0$ with $A_\mu = (-z\mathcal{E}, 0, 0, 0)$ given by

$$\Delta E = -\tfrac{1}{2}\alpha_s \mathcal{E}^2 \tag{3.24}$$

The shift in soliton energy (3.24) is obtained by substituting the field $A_\mu = (-z\mathcal{E}, 0, 0, 0)$ into the interaction part of the Lagrangian density; the seagull contribution to the static electric polarizability is obtained in the form

$$\alpha_s = \frac{e^2}{2} \int d^3\mathbf{x}\, z^2(F_\pi^2 \operatorname{Tr}(Q - U^+QU)^2 + \tfrac{1}{3}(F_K^2 - F_\pi^2)$$
$$\times \operatorname{Tr}\{(1 - \sqrt{3}\lambda_8)[(Q - U^+QU)^2U^+ + U(Q - U^+QU)^2]\}) \quad (3.25)$$

The dispersive contribution α_D is believed to be much smaller than α_s (Gobbi *et al.*, 1996) and will therefore not be taken into account in the present paper. It should also be noted that there is no contribution from the Wess–Zumino term (3.5), because of the properties of the completely antisymmetric tensor $e^{\mu\nu\alpha\beta}$. Introducing the adiabatically rotated bound-state ansatz into (3.25), we obtain

$$\alpha_S = [\alpha_1[1 - \tfrac{1}{2}(R_{33})^2] + |S|[\alpha_2 + \alpha_3(R_{33})^2] + \alpha_4 J_a^K R_{3a} + \alpha_5 J_3^K R_{33}] \quad (3.26)$$

where

$$\alpha_1 = \frac{4\pi}{15} e^2 F_\pi^2 \epsilon^5 \int_1^\infty dy\, y^4 \sin^2 F \quad (3.27)$$

$$\alpha_2 = \frac{1}{15} e^2 \epsilon^5 \int_1^\infty dy\, y^4 k(y)^2 (1 + 4\cos^2 F) \quad (3.28)$$

$$\alpha_3 = \frac{2}{15} e^2 \epsilon^5 \int_1^\infty dy\, y^4 k(y)^2 \sin^2 F \quad (3.29)$$

$$\alpha_4 = -\frac{2}{15} e^2 \epsilon^5 \int_1^\infty dy\, y^4 k(y)^2 (1 - 4\cos F) \quad (3.30)$$

$$\alpha_5 = -\frac{8}{15} e^2 \epsilon^5 \int_1^\infty dr\, y^4 k(y)^2 \cos^2 \frac{F}{2} \quad (3.31)$$

where ϵ is the constant cutoff defined by

$$\epsilon = \left\{\frac{3}{2}\frac{1}{ab}[cJ(J+1) + (1-c)I(I+1) + \frac{1}{4}c(c-1)|S|(|S|+2)]\right\}^{1/4} \quad (3.32)$$

and R_{ab} are the rotation matrices defined by

$$R_{ab} = \tfrac{1}{2}\operatorname{Tr}[\tau_a A\, \tau_b A^+] \quad (3.33)$$

Table I. Electric Polarizabilities of Bound-State Octet Hyperons

Particle	α_s
Λ	$\frac{5}{6}\alpha_1(\Lambda) + \alpha_2(\Lambda) + \frac{1}{3}\alpha_3(\Lambda)$
$\Sigma^{\pm}$	$\frac{5}{6}\alpha_1(\Sigma) + \alpha_2(\Sigma) + \frac{1}{3}\alpha_3(\Sigma) \pm \frac{1}{2}[\alpha_4(\Sigma) + \frac{1}{3}\alpha_5(\Sigma)]$
Σ^0	$\frac{5}{6}\alpha_1(\Sigma) + \alpha_2(\Sigma) + \frac{1}{3}\alpha_3(\Sigma)$
$\Xi^{\frac{0}{-}}$	$\frac{5}{6}\alpha_1(\Xi) + 2\alpha_2(\Xi) + \frac{2}{3}\alpha_3(\Xi) \pm \frac{2}{3}[\alpha_4(\Xi) + \frac{1}{3}\alpha_5(\Xi)]$

From (3.27)–(3.31) and (3.32) we see that elementary polarizabilities α_i ($i = 1, \ldots, 5$) are no longer universal for all bound-state octet hyperons, as in the case of the complete Skyrme model. The situation is similar to that of the constant-cutoff approach to magnetic moments of octet hyperons (Dalarsson *et al.*, 1993, 1995a–d, 1996a–c, 1997). The elementary polarizabilities are therefore different for different particle families (Λ, Σ, and Ξ). Using now the standard angular momentum techniques, following Gobbi *et al.* (1996), we obtain the static electric polarizabilities of ground-state octet baryons given in Table I.

3.4. Static Magnetic Polarizability

The static magnetic polarizability is extracted from the result for the shift in soliton energy in an external constant magnetic field $\mathbf{B} = B\mathbf{z}_0$ with $A_\mu = (0, \frac{1}{2}\mathbf{B} \times \mathbf{r})$ given by

$$\Delta E = -\tfrac{1}{2}\beta_S B^2 \tag{3.34}$$

The shift in soliton energy (3.34) is obtained by substituting the field $A_\mu = (0, \frac{1}{2}\mathbf{B} \times \mathbf{r})$ into the interaction part of the Lagrangian density, and now we must take into account both seagull and dispersive contributions. The quadratic part of the action (3.6) gives the seagull contribution in the form

$$\beta_S = -\frac{e^2}{8}\int d^3\mathbf{x}\,(r^2 - z^2)(F_\pi^2\,\mathrm{Tr}(Q - U^+QU)^2 + \tfrac{1}{3}(F_K^2 - F_\pi^2) \times \mathrm{Tr}\,\{(1 - \sqrt{3}\lambda_8)\,[(Q - U^+\,QU)^2\,U^+ + U(Q - U^+QU)^2]\}) \tag{3.35}$$

Introducing the adiabatically rotated bound-state ansatz into (3.35), we obtain

$$\beta_S = \beta_1[1 + \tfrac{1}{3}(R_{33})^2] + |S|[\beta_2 + \beta_3(R_{33})^2] + \beta_4 J_a^K R_{3a} + \beta_5 J_3^K R_{33} \tag{3.36}$$

where

$$\beta_1 = -\tfrac{3}{8}\alpha_1 \tag{3.37}$$

$$\beta_2 = -\tfrac{1}{2}\alpha_2 - \tfrac{1}{4}\alpha_3 \tag{3.38}$$

$$\beta_3 = \tfrac{1}{4}\alpha_3 \tag{3.39}$$

$$\beta_4 = -\tfrac{1}{2}\alpha_4 - \tfrac{1}{4}\alpha_5 \tag{3.40}$$

$$\beta_5 = \tfrac{1}{4}\alpha_5 \tag{3.41}$$

and α_i ($i = 1, \ldots, 5$) is given by (3.27)–(3.31). Using the same techniques as in the case of the static electric polarizabilities in Table I, we obtain the seagull contributions to the static magnetic polarizabilities of ground-state octet baryons given in Table II.

The linear part of the action (3.6), using second-order perturbation theory, gives a dispersive contribution to the static magnetic polarizability in the form

$$\beta_D = -\frac{e^2}{2M_N^2} \sum_{H \neq H'} \frac{|\langle H|\mu_H|H'\rangle|^2}{m_H - m_{H'}} \tag{3.42}$$

where H and H' refer to different hyperon states, and where μ_H is the magnetic moment operator of the hyperons, given in Dalarsson (1993, 1995a–d, 1996a–c, 1997) as

$$\mu_H = \mu_1 J_3^c - 2(\mu_2 - \mu_3|S|)R_{33} + \mu_4 J_3^K \tag{3.43}$$

Following Dalarsson (1993, 1995a–d, 1996a–c, 1997)

$$\mu_1 = -\frac{2M_N}{3\pi\Omega}\epsilon^2 \int_1^\infty dy\, y^2 \sin^2 F \frac{dF}{dy} \tag{3.44}$$

$$\mu_2 = \tfrac{1}{2}M_N\Omega \tag{3.45}$$

$$\mu_3 = \frac{1}{3} M_N \epsilon^3 \int_1^\infty dy\, y^2 k(y)^2 \cos^2 \frac{F}{2}\left(1 - 4\sin^2 \frac{F}{2}\right) \tag{3.46}$$

$$\mu_4 = c\mu_1 - \frac{4}{3} M_N \epsilon^3 \int_1^\infty dy\, y^2 k(y)^2 \cos^2 \frac{F}{2} \tag{3.47}$$

Table II. Seagull Magnetic Polarizabilities of Bound-State Octet Hyperons

Particle	β_s
Λ	$\frac{10}{9}\beta_1(\Lambda) + \beta_2(\Lambda) + \frac{1}{3}\beta_3(\Lambda)$
$\Sigma^\pm$	$\frac{10}{9}\beta_1(\Sigma) + \beta_2(\Sigma) + \frac{1}{3}\beta_3(\Sigma) \pm \frac{1}{2}[\beta_4(\Sigma) + \frac{1}{3}\beta_5(\Sigma)]$
Σ^0	$\frac{10}{9}\beta_1(\Sigma) + \beta_2(\Sigma) + \frac{1}{3}\beta_3(\Sigma)$
$\Xi^{\frac{0}{-}}$	$\frac{10}{9}\beta_1(\Xi) + 2\beta_2(\Xi) + \frac{2}{3}\beta_3(\Xi) \pm \frac{2}{3}[\beta_4(\Xi) + \frac{1}{3}\beta_5(\Xi)]$

Table III. Electric Polarizabilities of Octet Hyperons (in 10^{-4} fm) with Only the Seagull Contributions Taken into Account

Particle	α_s	α_s Set I[a]	α_s Set II[a]
Λ	15.8	18.1	28.0
Σ^+	17.1	18.8	29.4
Σ^-	15.2	17.4	26.5
Σ^0	16.2	18.1	28.0
Ξ^0	18.1	19.9	31.1
Ξ^-	16.7	18.0	27.3

[a]From Gobbi *et al.* (1996).

with (Dalarsson, 1993, 1995a–d, 1996a–c, 1997)

$$\Omega = \frac{2\pi}{3} F_\pi^2 \epsilon^3 \int_1^\infty dy\, y^2 \sin^2 F \tag{3.48}$$

$$c = 1 - \frac{8}{3} \omega \epsilon^3 \int_1^\infty dy\, y^2 k(y)^2 \cos^2 \frac{F}{2} \tag{3.49}$$

3.5. Numerical Results for Hyperon Polarizabilities

The numerical results for the electric and magnetic hyperon polarizabilities based on the model presented in the previous two sections, as well as the comparisons with the corresponding results obtained using the complete Skyrme model (Gobbi *et al.*, 1996), are given in Tables III and IV, respectively.

From Tables III and IV we see that the present static electric and magnetic hyperon polarizabilities are close to those obtained from the complete Skyrme model (Gobbi *et al.*, 1996) for $F_\pi = 186$ MeV, while there are more significant differences compared to the results obtained in Gobbi *et al.* (1996) for

Table IV. Magnetic Polarizabilities of Octet Hyperons (in 10^{-4} fm)

Particle	β_s	β_D	β_{TOT}	β_{TOT} Set I[a]	β_{TOT} Set II[a]
Λ	−7.9	10.4	2.5	3.4	−1.3
Σ^+	−8.5	9.5	1.0	1.3	−3.9
Σ^-	−7.5	0.4	−7.1	−7.9	−12.5
Σ^0	−8.0	−3.6	−11.6	−12.7	−17.3
Ξ^0	−7.5	11.4	3.9	4.4	−1.8
Ξ^-	−6.2	1.1	−5.1	−7.2	−12.4

[a]From Gobbi *et al.* (1996).

$F_\pi = 108$ MeV. However, as order-of-magnitude estimates, the present results are relatively accurate.

4. CONCLUSIONS

We have shown the possibility of using the Skyrme model to calculate the electromagnetic polarizabilities of hyperons without the use of the Skyrme stabilizing term, proportional to e^{-2}, which makes both the analytic and numerical treatment more difficult.

For such a simple model with only one arbitrary dimensional constant F_π, which is chosen equal to its empirical value $F_\pi = 186$ MeV, the accuracy in the prediction of the static electric and magnetic polarizabilities of hyperons is rather satisfactory. The results are close to those obtained from the complete Skyrme model (Gobbi *et al.*, 1996) for $F_\pi = 186$ MeV, while there are more significant differences compared to the results obtained in Gobbi *et al.* (1996) for $F_\pi = 108$ MeV. However, as order-of-magnitude estimates, the present results are relatively accurate.

Finally, it should be noted that in the present paper we have assumed, as in Gobbi *et al.* (1996), that the seagull contributions to the Hamiltonian are equal to the seagull contributions to the Lagrangian with opposite sign. A rigorous justification for such a simple prescription can be found in Dalarsson (1993, 1995a–d, 1996a–c, 1997).

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